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Characterization of a Point on Surface
- Proposal of Neighbourhood Function -

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1 Introduction

Three-dimensional geologic modelling is very effective method to express geologic structure under the ground. The algorithm and implementing methodology for 3-D geologic model using GRASS GIS and construction and visualization of 3-D geologic voxel model based on the logical model of geologic structure have been presented [1][2]. Geologic boundary surfaces and geologic boundary lines are important elements which compose 3-D geologic model. The mathematical expression of geologic boundary surfaces and lines based on the logical model of geologic structure has been presented [3]. As another approach, a concept of geologic units neighbouring around a point on surface and a function which assigns geologic units neighbouring around a point to the point, which is termed the neighbourhood function, are introduced to extract specific geologic boundary surface and line. If a point on surface is characterized by geologic units neighbouring around it, points on the surface can be classified as a group of points which compose a specific geologic boundary surface. An algorithm for finding geologic units neighbouring around a point on surface by the neighbourhood function is presented. Further, points on surface are classified as a group of points which compose specific geologic boundary surface and line using a simple geologic structure.

2 Basic theory

2.1 Logical model of geologic structure and geologic function

Let a 3-D subspace Ω be a survey area and suppose that the area Ω is composed of n geologic units that are disjoint:

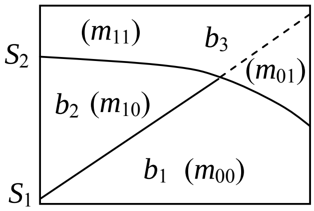
$$b_1 \cup b_2 \cup \cdots \cup b_n = \Omega \ ,$$
$$b_i \cap b_j = \phi \ \ (i \neq j).$$

A geologic function g which assigns a unique geologic unit to every point in the 3-D space Ω has been introduced to realize a 3-D geologic visualization in the GIS environment [1] [2].

$$g : \Omega \rightarrow B, \quad \text{where } B = \{ b_1, b_2, \ldots, b_n \} \ .$$

A simple geologic structure of Fig. 1(a) is used to explain fundamentals of the geologic function g. Three geologic units b_1, b_2 and b_3 are defined by two boundary surface S_1 and S_2 which divide Ω into two subspaces as follow;

(a)



(b)

	S_1	S_2
b_1	0	0
b_2	1	0
b_3	$*$	1

(c)

minset	unit
m_{00}	b_1
m_{01}	b_3
m_{10}	b_2
m_{11}	b_3

Figure 1: Basic elements of a geologic model. (a) relation between geologic units and surfaces in geologic section, (b) logical model (1 ; S_i^+ , 0 ; S_i^- , $*$; no specific relation with the surface.), (c) relational code table.

$$b_1 = S_1^- \cap S_2^- , \quad b_2 = S_1^+ \cap S_2^- , \quad b_3 = S_2^+ ,$$

where S_i^+ and S_i^- give subspaces that lie above and below the surface S_i, respectively. These equations can be expressed in a tabular form as shown in Fig. 1(b). The above equations and table define the relation between geologic units and boundary surfaces. This logical relation is termed “the logical model of geologic structure” [4].

The geologic units b_1, b_2, …, b_n can be expressed in a “*minset standard form*” [5]. The *minset* is a minimum subspace that is divided by the boundary surfaces S_1, S_2, …, S_p in the 3-D space Ω. Let $m_{d1d2...dp}$ be a *minset* defined by;

$$m_{d1d2...dp} = h_1(d_1) \cap h_2(d_2) \cap \ldots \cap h_p(d_p) ,$$

$$\text{where } h_i(d_i) = \begin{Bmatrix} S_i^+ ; d_i = 1 \\ S_i^- ; d_i = 0 \end{Bmatrix} .$$

In the case of Fig. 1(a), four *minsets* can be defined as follows;

$$m_{00} = S_1^- \cap S_2^- , \quad m_{01} = S_1^- \cap S_2^+ , \quad m_{10} = S_1^+ \cap S_2^- , \quad m_{11} = S_1^+ \cap S_2^+ .$$

The relation between *minset standard forms* and geologic units is shown as follows;

$$b_1 = m_{00} , \quad b_2 = m_{10} , \quad b_3 = m_{01} \cup m_{11} .$$

Each *minset* is included in only one of geologic units as shown below;

$$m_{00} \subset b_1 , \quad m_{01} \subset b_3 , \quad m_{10} \subset b_2 , \quad m_{11} \subset b_3 .$$

The relation between *minsets* and geologic units can be expressed by a function g_1 from a class of *minsets* M into B:

$$g_1 : M \to B .$$

The function g_1 can be represented by the relational code table shown in Fig. 1(c).

Further, for a point $P(x, y, z)$ in a space Ω, a *minset* $m_{d1d2...dp}$ can be assigned a value of d_i = 1 or d_i = 0 depending on whether $P(x, y, z)$ falls in S_i^+ or S_i^-, respectively. This correspondence between every point in Ω and *minsets* is expressed by a function g_2:

$$g_2 : \Omega \to M .$$

Consequently, a convolution of functions g_1: $M \to B$ and g_2: $\Omega \to M$ provides a rule to define the geologic unit that includes a given point $P(x, y, z)$:

$$g\,(x, y, z) = g_1\,(\,g_2\,(x, y, z)) .$$

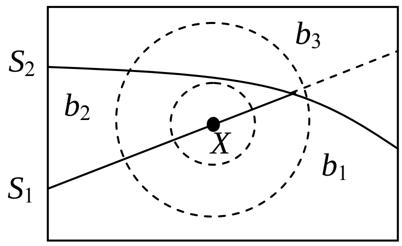


Figure 2: Neighbourhood function of function g.

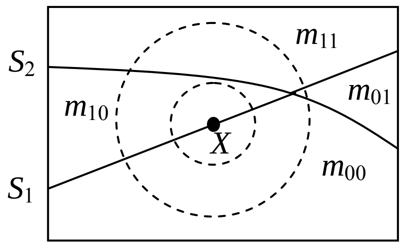


Figure 3: Neighbourhood function of function g_2.

	Surface S_1	Surface S_2
Point X	$*$	0

Table 1: Relation between point X and surfaces.

2.2 Neighbourhood function

An open ball of radius ε centred at point X is termed ε-neighbourhood of X. This is denoted by $V(X, \varepsilon)$. A image of $V(X, \varepsilon)$ by the geologic function g which assigns a unique geologic unit to every point in the 3-D space Ω shows a set of geologic units in ε-neighbourhood of X:

$$g\,(V\,(X, \varepsilon)) = \{g(x) : x \in V\,(X, \varepsilon)\}\ .$$

As the radius ε becomes small, the number of geologic units in $V(X, \varepsilon)$ decreases (Fig. 2). When $g\ (V(X, \varepsilon))$ becomes the same set for all the smaller radii ε than a certain value, $g(V(X, \varepsilon))$ is called "geologic units neighbouring around X" and is denoted by $\min g\ (V\ (X, \varepsilon))$.
A function $G\colon \Omega \to 2^B$ which assigns $\min g\ (V(X, \varepsilon))$ to point X is termed "neighbourhood function of function g" if $\min g\ (V\ (X, \varepsilon))$ exists for all points X in the 3-D space Ω.

The neighbourhood function can be expanded to various functions. Similarly, the neighbourhood function of the function g_2 can be defined (Fig. 3). A function $G_2 : \Omega \to 2^M$ which assigns "*minsets* neighbouring around X" to X is called "neighbourhood function of function g_2".

3 Geologic units neighbouring around a point

The geologic units neighbouring around a point on surface can be found using the neighbourhood function of function g_2 and the function g_1. As an example, steps for finding geologic units neighbouring around the point X in Fig. 2 are shown below.

Step 1; generate the relational code table (the function g_1) from the logical model of geologic structure (Fig. 1(c)).

Step 2; obtain the relation of the height between the point X and every boundary surface. If the point X is higher than a surface, the relation is expressed by "1". If lower then the relation is "0", if on the surface then the relation is "$*$". The relation between the point X and surfaces is shown in Table 1.

Step 3; find *minsets* neighbouring around the point X using the relation of the height between the point X and every boundary surface. The relation between a point and surfaces corresponds to subscript $d_1d_2...d_p$ of *minsets* neighbouring around the point. When the point X is higher than the surface S_i, the *minsets* neighbouring around the point

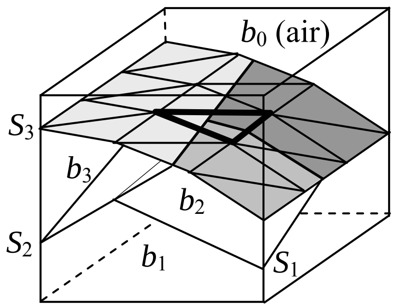


Figure 4: Geologic model.

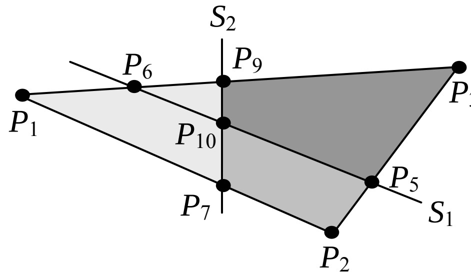


Figure 5: Triangular mesh.

b_0 (air)	*	*	1
	S_1	S_2	S_3
b_1	0	0	0
b_2	1	0	0
b_3	*	1	0

Table 2: Logical model of geologic structure.

minset	unit
m_{000}	b_1
m_{001}	b_0
m_{010}	b_3
m_{011}	b_0
m_{100}	b_2
m_{101}	b_0
m_{110}	b_3
m_{111}	b_0

Table 3: Relational code table.

is also in upper side of the surface. Therefore, d_i is "1" when the relation between the point X and surface S_i is "1". Similarly, d_i is "0" when the relation between the point X and surface S_i is "0". When the point X is on the surface S_i, the *minsets* neighbouring around the point exist in both the upper side and lower side of the surface. Therefore, d_i is both "1" and "0". According to the rule above, the *minsets* neighbouring around the point X in Fig. 2 are m_{00} and m_{10}.

Step 4; derive geologic units from *minsets* using the relational code table. In the case of this example, the geologic units neighbouring around the point X are b_1 and b_2.

4 Classification of points on surface

Using a geologic structure composed of four geologic units as shown in Fig. 4, geologic units neighbouring around a point on surface are obtained. Three boundary surfaces S_1, S_2 and S_3 are grid data and the logical model of geologic structure is given in Table 2.
First, the relational code table is prepared from the logical model of geologic structure (Table 3). Secondly, each grid cell is divided into two triangular meshes and following process is implemented in each triangular mesh. An example of the triangular mesh surrounded by a thick line in Fig. 4 is shown (Fig. 5).
A number is assigned to points on surface and a table like Table 4 is prepared to obtain geologic units neighbouring around the points. P_1, P_2 and P_3 are assigned to each of triangular vertexes. P_4, P_5 and P_6 are given to each of points where three sides of the triangle and surface S_1 cross. Similarly, the intersection points between three sides of the

	Coordinates			Triangular sides			Intersection line between surface and triangle		Relation of height			Geologic units in the neighbourhood				
	x	y	z	P_1P_2	P_2P_3	P_1P_3	$S_1 \cap \Delta$	$S_2 \cap \Delta$	S_1	S_2	S_3	b_0	b_1	b_2	b_3	
P_1				1		1			1	1	*	1			1	
P_2				1	1				1	0	*	1		1		
P_3					1	1			0	0	*	1	1			
P_4				1			1									outside
P_5					1		1		*	0	*	1	1	1		
P_6						1	1		*	1	*	1			1	
P_7				1				1	1	*	*	1		1	1	
P_8					1			1								outside
P_9						1		1	0	*	*	1	1		1	
P_{10}							1	1	*	*	*	1	1	1	1	

Table 4: Geologic units neighbouring around a point on surface.

Geologic boundary	Component
b_0/b_1 surface	P_3, P_5, P_9, P_{10}
b_0/b_2 surface	P_2, P_5, P_7, P_{10}
b_0/b_3 surface	$P_1, P_6, P_7, P_9, P_{10}$
$b_0/b_1/b_2$ line	P_5, P_{10}
$b_0/b_1/b_3$ line	P_9, P_{10}
$b_0/b_2/b_3$ line	P_7, P_{10}

Table 5: Relation between geologic boundary and points.

triangle and surface S_2 are P_7, P_8 and P_9, respectively. P_{10} is assigned to the point where S_1, S_2 and triangle intersect. In Table 4, "The relation between point and triangular sides" and "the relation between point and intersection line between surface and triangle" are the information which shows the position of the point. For example, if a point is on P_1P_2, the relation between the point and P_1P_2 is "1" and if a point is on the intersection line between S_1 and the triangular mesh, the relation between the point and $S_1 \cap \Delta$ is "1".

The coordinates of each point are calculated and geologic units neighbouring around the point are obtained by the proposed algorithm. P_4 and P_8 are outside of the triangular mesh in the case of this example.

Using the completed table, the points on surface can be classified as a group of points which compose specific geologic boundary surface and line (Table 5). For example, the points which compose the geologic boundary surface between b_0 and b_1 are P_3, P_5, P_9 and P_{10}, which have b_0 and b_1 in the neighbourhood. The points which compose the geologic boundary line between b_1 and b_2 on the topographic surface S_3 are P_5 and P_{10}, which have b_0, b_1 and b_2 in the neighbourhood.

5 Conclusion

In the present work, a concept of geologic units neighbouring around a point on surface and a neighbourhood function which assigns geologic units neighbouring around a point to the point have been introduced to extract specific geologic boundary surface and line. The neighbourhood function can be expanded to not only the geologic function but also various functions.

The algorithm for finding geologic units neighbouring around a point on surface was developed. The geologic units neighbouring around a point on surface are obtained using the neighbourhood function of function g_2 and the function g_1. The points characterized by geologic units in the neighbourhood can be classified as a group of points which compose a specific geologic boundary surface.

In the future, an algorithm for constructing geologic boundary surfaces and lines from extracted groups need to be developed to visualize a specific geologic boundary and utilize it for various analyses.

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